



Description Logic

Summer Semester 2021

Exercise Sheet 9

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Prof. Dr.-Ing. Franz Baader, Dr. Oliver Fernández Gil

Exercise 9.1 Extend the $\mathcal{ALC}$ -Elim algorithm to the description logic $\mathcal{ALCI}$. Prove the correctness of the extended algorithm.

Exercise 9.2 Determine whether Player 2 has a winning strategy in the following infinite Boolean games, where the initial configuration t_0 assigns *false* to all variables:

- (a) $\varphi = (p_1 \wedge p_2 \wedge \neg q_1) \vee (p_3 \wedge p_4 \wedge \neg q_2) \vee (\neg(p_1 \vee p_4) \wedge q_1 \wedge q_2)$ with $\Gamma_1 = \{p_1, p_2, p_3, p_4\}$ and $\Gamma_2 = \{q_1, q_2\}$
- (b) $\varphi = ((p_1 \leftrightarrow \neg q_1) \wedge (p_2 \leftrightarrow \neg q_2) \wedge (p_1 \leftrightarrow p_2)) \vee ((p_1 \leftrightarrow q_1) \wedge (p_2 \leftrightarrow q_2) \wedge (p_1 \leftrightarrow \neg p_2))$ with $\Gamma_1 = \{p_1, p_2\}$ and $\Gamma_2 = \{q_1, q_2\}$

Exercise 9.3 Are the following variations of infinite Boolean games also ExpTime-hard?

- (a) Player 1 wins if the constructed truth assignment falsifies the formula φ , instead of satisfying it.
- (b) Player 2 starts instead of Player 1.
- (c) The variables are not assigned to a specific player; instead, the active player can choose any variable and assign it a new truth value; variables can be chosen multiple times.

Exercise 9.4 The *universal role* is a role name u whose interpretation is fixed as $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ in any interpretation $\mathcal{I}$. Let $\mathcal{ALC}^u$ denote the extension of $\mathcal{ALC}$ with the universal role. Show that concept satisfiability in $\mathcal{ALC}^u$ without TBoxes is ExpTime-complete.